

The Complex Dimensions of Self-Similar Subsets of p -adic Product Spaces

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Graduate Student Seminar

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Definitions & Notation

Homogeneous measures

The distance zeta function

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Definitions & Notation



Definitions & Notation: Homogeneous measures

Let (X, d, μ) be a complete, separable metric measure space such that

$$0 < \mu(B(x, r)) < \infty$$

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Definition

We say that μ is *q-homogeneous* on A if there is some constant $M > 0$ such that

$$\frac{\mu(B(x, r))}{\mu(B(\xi, \rho))} \leq M \left(\frac{r}{\rho}\right)^q$$

for all $0 < \rho < r \leq \text{diam}(A)$, all $x \in A$, and all $\xi \in B(x, r)$.

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for all $0 < \rho < r \leq \text{diam}(A)$, all $x \in A$, and all $\xi \in B(x, r)$. The *measure theoretic Assouad dimension* of A is

$$\dim_{\text{As}}(A) := \inf \{q \geq 0 \mid \mu \text{ is } q\text{-homogeneous on } A\}.$$

Definitions & Notation: The distance zeta function

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Suppose that $\dim_{\text{As}}(X) = Q$ and that A is a bounded subset of X . For $\delta > 0$, define

$$A_\delta := \{x \in X \mid d(x, A) \leq \delta\}.$$



Definitions & Notation: The distance zeta function

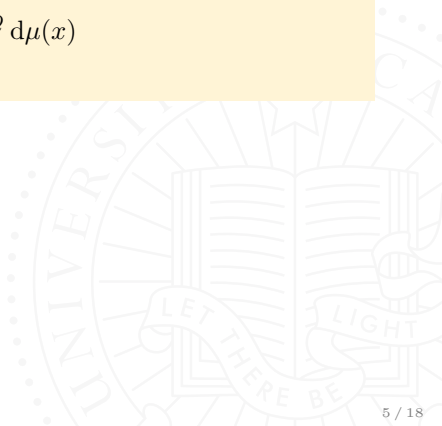
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Suppose that $\zeta_A(s)$ can be meromorphically extended to a (strictly) larger domain. Then the *complex dimensions* of A , denoted $\mathcal{P}(A)$, are the poles of this extension. That is

$$\mathcal{P}(A) := \{\omega \in \mathbb{C} \mid \omega \text{ is a pole of } \zeta_A(s)\}.$$

Definitions & Notation: p -adic spaces

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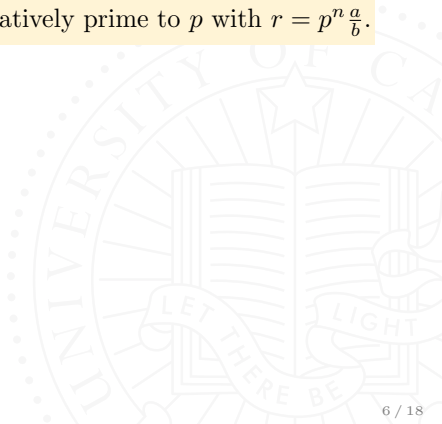
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Let $r \in \mathbb{Q}$. The *p -adic absolute value* of r is given by

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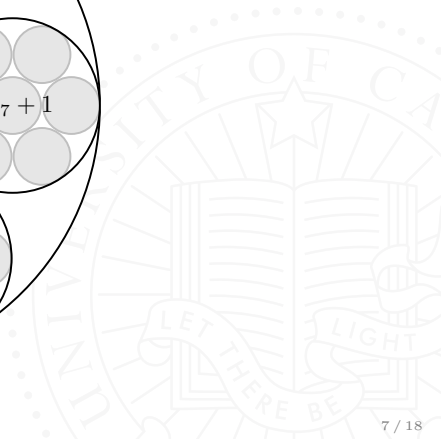
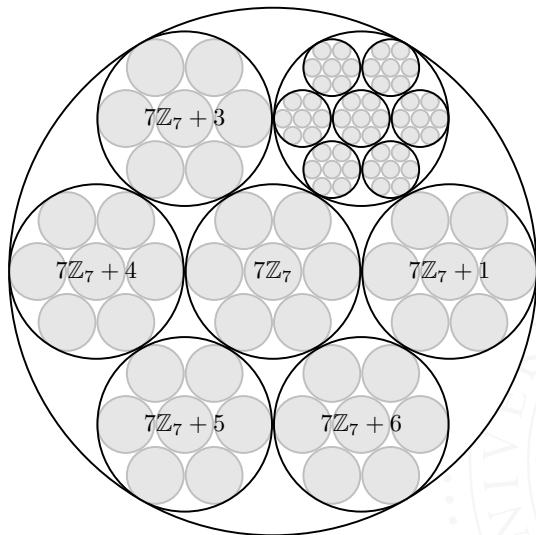
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\mathbb{Q}_p is equipped with the Haar measure μ such that $\mu(\mathbb{Z}_p) = 1$.

Definitions & Notation: p -adic spaces



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Let $Q \in \mathbb{N}$ and $\alpha \in [1, \infty)$.

Notation

On the product space \mathbb{Q}_p^Q , define the equivalent metrics

$$d^\alpha(\mathbf{x}, \mathbf{y}) := \left(\sum_{i=1}^Q |x_i - y_i|_p^\alpha \right)^{1/\alpha},$$

and

$$d^\infty(\mathbf{x}, \mathbf{y}) := \max \left\{ |x_i - y_i|_p \mid 1 \leq i \leq Q \right\}.$$

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Lemma

For any $Q \in \mathbb{N}$ and any $\alpha \in [1, \infty]$, the product space $(\mathbb{Q}_p^Q, d^\alpha, \mu)$ satisfies

$$\dim_{\text{As}}(\mathbb{Q}_p^Q) = Q,$$

where μ is the natural product measure.

Definition

A *self-similar iterated function system* (SSIFS) on \mathbb{Q}_p^Q is a finite collection of maps $\{\varphi_j\}_{j \in \mathcal{J}}$, each of which is of the form

$$\varphi_j(x) = p^{k_j} x + b_j,$$

where $k_j \in \mathbb{N}$ and $b_j \in \mathbb{Q}_p^Q$.

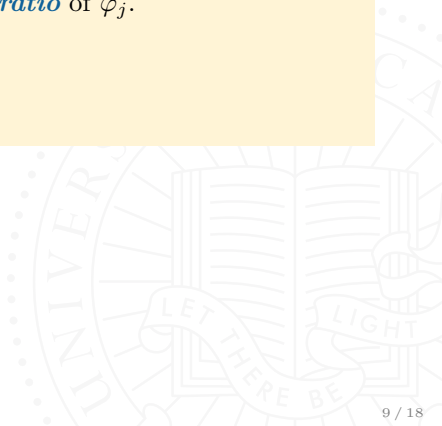


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$$\Phi(E) := \bigcup_{j \in \mathcal{J}} \varphi_j(E).$$

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Theorem

Let Φ be as above. Then there is a unique, nonempty, compact set $\mathcal{A} \subseteq \mathbb{Q}_p^Q$ such that

$$\Phi(\mathcal{A}) = \mathcal{A}.$$

We call \mathcal{A} the *attractor* of the SSIFS.

Definitions & Notation: Iterated function systems on \mathbb{Q}_p^Q

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Let \mathcal{J}^* denote the set of all finite sequences (or “words”) with entries in \mathcal{J} . For each

$$J = (j_1, j_2, \dots, j_n) \in \mathcal{J},$$

define

$$\varphi_J = \varphi_{j_n} \circ \varphi_{j_{n-1}} \circ \dots \circ \varphi_1.$$

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Let $\omega = () \in \mathcal{J}^*$ denote the “empty word.” We adopt the convention that φ_ω is the identity map, i.e.

$$\varphi_\omega(x) = x.$$

Results & Examples



Theorem

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Theorem

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$$\zeta_{\mathcal{A}}(s) = \zeta_{\mathcal{A}, \Omega_t}(s) \sum_{n=0}^{\infty} C_n p^{-ns},$$

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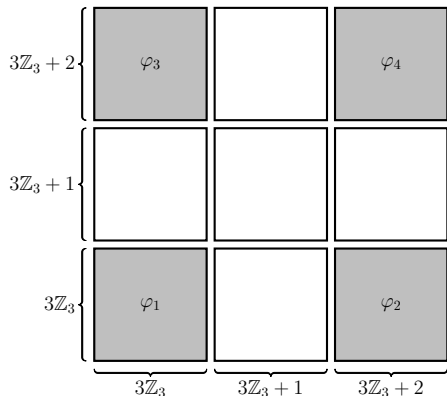
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and C_n counts the number of maps of the form φ_J for some $J \in \mathcal{J}^*$ with contraction ratio p^{-n} .

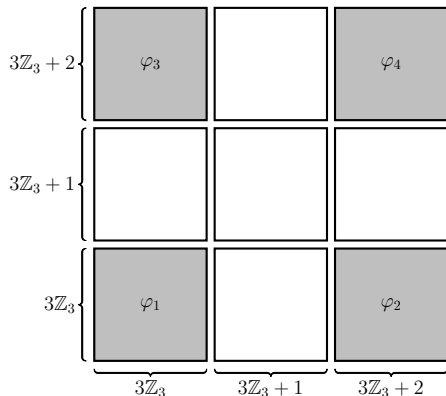
Results & Examples: 3-adic Cantor dust



Example

Let $\{\varphi_j\}_{j=1}^4$ be the SSIFS on \mathbb{Q}_3^2 that maps \mathbb{Z}_3^2 into the four rectangles shown to the left. Let \mathcal{C}^2 denote the attractor of this SSIFS.

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We may also regard \mathcal{C}^2 as the Cartesian product of two copies of a 3-adic Cantor set. In either case, \mathcal{C}^2 is an analog of the ternary Cantor dust in \mathbb{R}^2 .

Example (con't)

With respect to d^∞ ,

$$\zeta_{\mathcal{C}^2, \Omega_i}(s) = \int_{\mathbb{Z}_3^2 \setminus \Phi(\mathbb{Z}_3^2)} d^\infty(x, \mathcal{C}^2)^{s-2} d\mu(x) = \mu(\mathbb{Z}_3^2 \setminus \Phi(\mathbb{Z}_3^2)) = \frac{5}{9}.$$

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Therefore

$$\mathcal{P}(\mathcal{C}^2) = \frac{\log(4)}{\log(3)} + i \frac{2\pi\mathbb{Z}}{\log(3)}.$$

Example

Fix a prime p and define maps on \mathbb{Q}_p by

$$\varphi_1(x) = px, \quad \text{and} \quad \varphi_2(x) = p^2x + 1.$$

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Results & Examples: Fibonacci attractors

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$$C_n = \frac{1}{\sqrt{5}} (\phi^{n+1} + \psi^{n+1}), \quad \text{where} \quad \phi, \psi = \frac{1 \pm \sqrt{5}}{2}.$$

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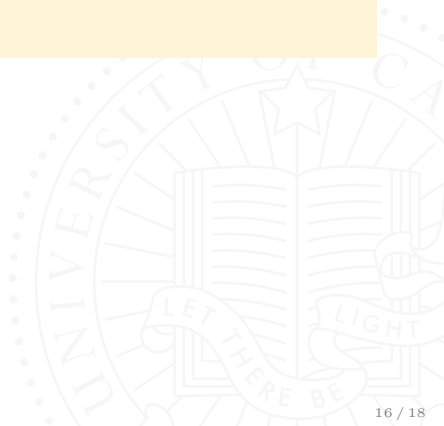
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Results & Examples: Fibonacci attractors

Example (con't)

And so

$$\zeta_{\mathcal{F}}(s) = \left(\frac{p-2}{p} + \frac{p-1}{p^2} p^{1-s} \right) \frac{\sqrt{5} p^{2s}}{(p^s - \phi)(p^2 - \psi)}.$$



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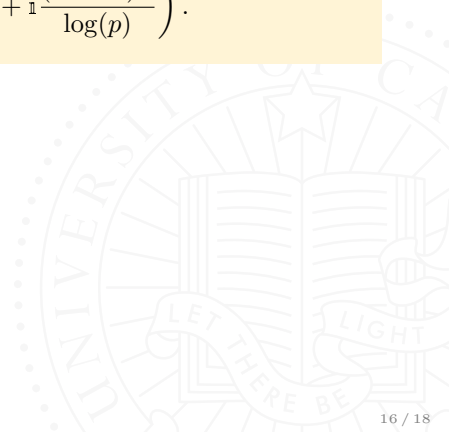
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Therefore

$$\mathcal{P}(\mathcal{F}) = \left(\frac{\log(\phi)}{\log(p)} + i \frac{2\pi\mathbb{Z}}{\log(p)} \right) \cup \left(-\frac{\log(\phi)}{\log(p)} + i \frac{(2\pi+1)\mathbb{Z}}{\log(p)} \right).$$



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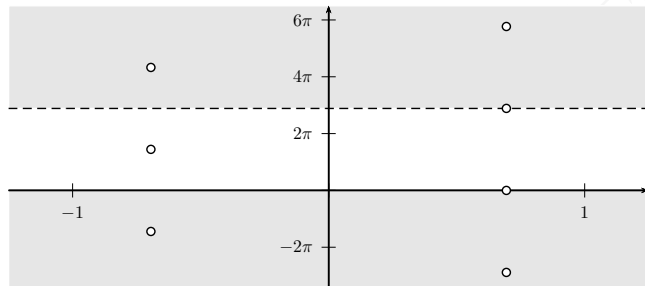
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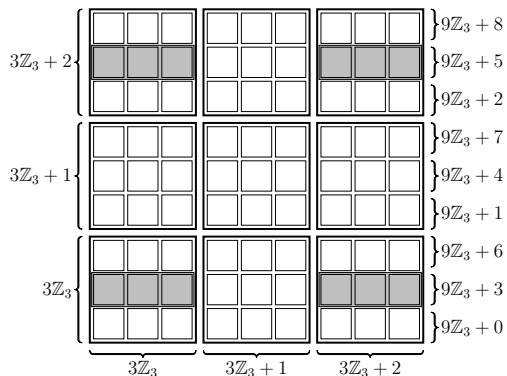
$$\zeta_{\mathcal{F}}(s) = \left(\frac{p-2}{p} + \frac{p-1}{p^2} p^{1-s} \right) \frac{\sqrt{5} p^{2s}}{(p^s - \phi)(p^2 - \psi)}.$$

Therefore

$$\mathcal{P}(\mathcal{F}) = \left(\frac{\log(\phi)}{\log(p)} + i \frac{2\pi\mathbb{Z}}{\log(p)} \right) \cup \left(-\frac{\log(\phi)}{\log(p)} + i \frac{(2\pi+1)\mathbb{Z}}{\log(p)} \right).$$



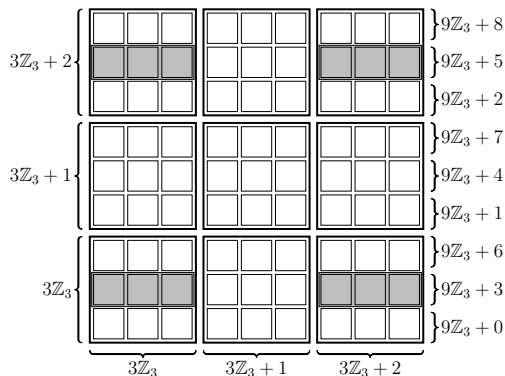
Results & Examples: A McMullen carpet analog



Example

Let \mathcal{A} denote the attractor of the IFS shown to the left.

Results & Examples: A McMullen carpet analog



Example

Let \mathcal{A} denote the attractor of the IFS shown to the left. With respect to d^∞ ,

$$\mathcal{P}(\mathcal{A}) = \left(\frac{3 \log(2)}{2 \log(3)} + i \frac{\pi \mathbb{Z}}{\log(3)} \right) \cup \left(\frac{\log(4)}{\log(3)} - 1 + i \frac{2\pi \mathbb{Z}}{\log(3)} \right).$$

Selected Bibliography

- [1] Michel L. Lapidus and Hùng Lũ', *Nonarchimedean cantor set and string*, J. Fixed Point Theory and Appl. **3** (2008), no. 1, 181–190.
- [2] Michel L. Lapidus, Goran Radunović, and Darko Žubrinić, *Fractal zeta functions and fractal drums*, Springer, 2017.
- [3] Curt McMullen, *The Hausdorff dimension of general Sierpński carpets*, Nagoya Mathematical J. **96** (1984), 1–9.

